MHD TURBULENCE AND STATISTICS OF ENERGY RELEASE IN THE SOLAR CORONA

M. Georgoulis\textsuperscript{1,3}, M. Velli\textsuperscript{2} and G. Einaudi\textsuperscript{1}

\textsuperscript{1}Dipartimento di Fisica, Università di Pisa, Italy
\textsuperscript{2}Dipartimento di Astronomia e Scienze dello Spazio, Università di Firenze, Italy
\textsuperscript{3}Department of Physics, University of Thessaloniki, Thessaloniki, Greece

\section*{ABSTRACT}

We present a statistical analysis of long-time simulations of a two-dimensional section of a coronal loop subject to random magnetic forcing within the framework of reduced magnetohydrodynamics (MHD). Even with a moderate magnetic Reynolds number, intermittency is clearly present both in space and time. The response of the loop to random forcing, as described either by the time series of the average and maximum energy dissipation or its spatial distribution at a given time, displays a gaussian noise component which may be subtracted to define discrete dissipative events. Distribution functions of both maximum and average current dissipation, of total energy content, of peak activity and of duration of such events are all shown to display robust scaling laws.

Key words: MHD turbulence; Nanoflares; Coronal Heating.

\section*{1. INTRODUCTION}

Parker (1972) was the first to suggest that coronal heating could be the necessary outcome of an energy flux associated with the tangling of coronal field lines by photospheric flows. Parker (1983, 1988), Sturrock and Uchida (1981), Van Ballegooijen (1986) and Berger (1991) among others further explored the dynamics caused by such random shuffling of magnetic field lines. Numerical simulations by Mikic et al. (1989) confirmed that this process causes a nonlinear cascade leading to an exponential growth of local coronal currents. In subsequent papers, Parker clarified this scenario for coronal heating by introducing the terms “microflare” and then “nanoflare” to describe the dissipation of elementary coronal current sheets developed as a consequence of the random footpoint motion (Parker 1989, 1991).

Observations of the statistical behaviour of flaring activity have brought some evidence in favour of the “nanoflare” coronal heating scenario. Flare distributions such as number of flares as a function of total energy content, number of flares as a function of peak luminosity and number of flares as a function of duration, all display well-defined power laws, extending over several orders of magnitude down to instrumental resolution. However, if such flaring activity is to account for coronal heating, it is necessary (Hudson 1991) for the power law distribution at energies below the observed lower limit to switch to a significantly steeper scaling index (i.e. the number of “nanoflares” per interval of energy should significantly increase at low energies). Although power-law behaviour of energy release has been modeled successfully using cellular automaton models (SOC) of magnetic field instabilities (Lu & Hamilton 1991, Lu et al. 1993) there has been no evidence for such behaviour coming either from MHD models or from the 3D numerical simulations reported up to now (Mikic et al. 1989, Strauss 1993, Longcope & Sudan 1994, Galsgaard & Nordlund 1996). Using fairly long-time simulations of magnetically forced MHD turbulence in a two dimensional slab geometry, chosen to model a section of a coronal loop, Einaudi et al. 1996 reported a number of results concerning the overall properties of energy release with varying resolution and Reynolds numbers; they demonstrated in particular that for a given random forcing, there is an increase in average dissipation with increasing Reynolds number as well as an increase of peak to average energy dissipation.

Einaudi et al.’s simulations lasted about 600 poloidal Alfven times, where the poloidal Alfvén time is given by the time it takes an Alfvén wave to cross the coronal loop given the typical value of the magnetic field in the plane of the simulation, and corresponds (as discussed in the following section) to about 10 times the transit time along the coronal loop.

Here we extend the simulations of Einaudi et al. to much longer times (4000 poloidal Alfvén times) since a very large data set is necessary to obtain reliable statistics. We show that the spatial average of the dissipated power displays non-gaussian statistics and that upon subtraction of the gaussian component a well-defined power law results for the number of “dissipative events” as a function of total dissipative energy; the same result also holds for the local peak dissipation as well as for the spatial distribution at a given time. This is to our knowledge the first direct connection between the behaviour of a system described by MHD and the observational distribution of power release in the solar corona. The main limitation of our model is that no net magnetic helicity is injected in our system, and the energy storage capability is much reduced as compared to a truly three-dimensional system forced at the boundaries.

\begin{flushright}
\end{flushright}

© European Space Agency • Provided by the NASA Astrophysics Data System
Our model probably does not therefore have much direct significance for the interpretation of large observed solar flares.

2. NUMERICAL MODEL

We model the cross-section of a coronal loop threaded by a large axial magnetic field $B_0$ with footpoints at either end rooted in the photosphere. Denoting by $z$ the axial direction, any disturbance originating in the photosphere propagates along $z$ with the associated Alfvén velocity and gives rise to perpendicular magnetic and velocity fields $\vec{b}_z$ and $\vec{u}_\perp$. The time evolution of the perpendicular fields is well described by the reduced MHD equations (Strauss 1976, 1977, Georgoulis et al. 1997), which are appropriate in a regime of: small ratio of kinetic to magnetic pressures; large loop aspect ratio ($\varepsilon \equiv l/L \ll 1, L$ being the length of the loop and $l$ the minor radius of the loop); small ratio of poloidal to axial magnetic field ($b_z/B_0 \ll \varepsilon$). Incompressibility implies constant density in each plane which allows the use of the same (velocity) units for velocity and magnetic fields via the normalization $b \rightarrow b_0/\sqrt{\varepsilon}$. Such equations are essentially the 2D incompressible MHD equations, with communication across planes being given by transverse Alfvén waves propagating with the axial Alfvén velocity, based on $B_0$. In the 3D problem, it is such terms which describe the photospheric forcing due to the advection of the axial field by photospheric velocity fields. In our 2D version, we model them according to our present knowledge of the photospheric driver: $B_0(\partial \vec{u}_\perp/\partial z) = \vec{F}_m(x,y,t)$, $B_0(\partial \vec{b}_z/\partial z) = \vec{F}_u(x,y,t)$, where $\vec{F}_m, \vec{F}_u$ are unknown forcing functions. In the 3D system, plasma pressure $\ll$ magnetic pressure implies that the magnetic field evolves in a sequence of force-free equilibria which also implies that large-scale kinetic energy $\ll$ magnetic free energy. We thus take $\vec{F}_u = 0$ in our two-dimensional simulations. Incompressibility of the flow means we may write $\vec{F}_m = \nabla \times \vec{f}_m$. We have performed numerical simulations with a forcing term $f_m$ which is a periodic random function with non vanishing Fourier components only in the "ring" of wavenumbers $3 \leq k \leq 4$. The forcing term consists of two "eddies" with a turnover time $2\tau^*$ and which are temporally out of phase. At the end of each $\tau^*$-interval the amplitude of each "eddy" is alternately changed randomly (with a uniform amplitude distribution over the interval $[0,1]$ and a uniform phase distribution over $[0,2\pi]$). The overall eddy amplitude is then renormalized so that the rms spatial non-dimensional value for the forcing term $f_m$ is also unity ($< f_m > = 1$). The physical units of the model are then fixed in terms of the large-scale magnetic field $B_0$, the typical photospheric velocity $u_{ph}$ (in units of $B_0$), the loop length $L$ and the aspect ratio $l/c$. Let us denote the units of magnetic field / velocity and length used to render our equations non-dimensional respectively with $b_0$ and $l_1$, along with the time unit $\tau = l_1/B_0$, which will be hereafter called a "timestep". A dimensional analysis leads then to

$$1 \sim < f_m > \sim \frac{B_0}{b_0} u_{ph} \frac{\tau}{L} \quad b_0 \sim B_0 \left( \frac{l_1 u_{ph}}{B_0 L^2} \right)^{\frac{1}{2}}.$$

$b_0$ is the poloidal Alfvén velocity is the only velocity unit directly relevant to our simulations, since the axial field does not appear in the equations. Assuming the photospheric velocity to satisfy $u_{ph} \simeq 0.001B_0$ and a value $1/\varepsilon = 10$ for the loop aspect ratio, we obtain

$$b_0 \simeq 0.01 \quad \tau \simeq 10 \frac{L}{B_0} \text{s}$$

If $B_0 \simeq 1000 \text{ km/s}$ and $L \simeq 10^4 \text{ km}$, $\tau$ turns out to be $\tau \simeq 100 \text{ s}$. Our numerical domain is a square mesh with periodic boundary conditions. We have performed a series of simulations with a resolution $128 \times 128$, $\tau^* = 16\tau$, extending from $t = 0$ up to $t = 4100$, that is $\sim 114 \text{ hr}$ real coronal time with the choice of constants given above. Resistivity and viscosity are adapted to the grid (in case of a $128 \times 128$ resolution the value used is $\eta = \nu = 0.01$). In addition, we have performed a limited number of higher-resolution runs, using a grid with dimensions $256 \times 256$ (the value of the resistivity used in this case is $\eta = \nu = 0.004$). The latter data set will be mostly used for statistical analysis in space, while an analysis of the temporal evolution of the system will be carried out for the lower-resolution case.

Figure 1. (a) Time evolution of the average current dissipation. The dashed line corresponds to the temporal noise threshold. (b) Low-energy part of the $< \eta J^2 >$ distribution function (solid line) and the best $\chi^2$-fit obtained (dashed line).
3. TEMPORAL EVOLUTION AND SPATIAL STRUCTURE

In Fig. 1a we show the time evolution of the current dissipation averaged over the whole system $E_D = <\eta J^2>$. The timeseries is characterized by a high level of intermittency, with an order of magnitude variation of the dissipated energy over short time scales. To quantitatively define a "dissipative event", we must define a threshold which allows one to define the beginning and end of an event. The data plotted in Fig. 1a have been used to build up a discrete distribution function of the dissipated power over the entire timeseries. The technique used to derive the distribution function from the data is described in Georgoulis et al. 1997. The resulting distribution function is shown by the solid line in Fig 1b. The noise component is well represented by a $\chi^2$-distribution function of the form

$$N(x) = \frac{C}{2\Gamma(\frac{n}{2})} \left(\frac{x}{\sigma}\right)^{\frac{n}{2}-1} \exp(-\frac{x}{\sigma})$$

where $x \equiv E_D$, $\Gamma(\frac{n}{2})$ is the Gamma function, $n$ is the number of degrees of freedom and $C$, $\sigma$ are the fitting constants. The best fit for our timeseries is obtained for $n = 17$ (dashed curve in Fig 1b). The $\chi^2$-confidence level of the fit is close to 100%. For $E_D \geq E_n = 1.6$ the measured distribution function begins to deviate significantly from the $\chi^2$-fit, which defines the temporal noise threshold of the average-current-dissipation timeseries. Having subtracted the background, we are able to identify a total number of 369 discrete events for the average current dissipation, a mean number of events above noise of about 3.2 hr$^{-1}$. The distribution functions of total energy content, peak activity and duration of such events are plotted in Figs 2a, 2b, 2c. They all may be represented by robust power laws extending over 2 - 2.5 orders of magnitude. Notice that events last from $\sim 1.6$ min ($\sim 1\tau$), up to $\sim 4.1$ hr (150$\tau$). The distribution function of the peak activity displays a power law over one order of magnitude. The reason is that the maximum current which can be obtained in the integration box is limited by the spatial resolution and decreases with increasing resistivity.

The scaling indices of the distribution functions are $\delta_T = \approx -1.32$ (for total energy content), $\delta_L = \approx -2.81$ (for peak activity) and $\delta_D = \approx -1.36$ (for duration). The average power dissipated in all events above noise is smaller than the average power dissipated in the background, the ratio $(S/N)_{av} \approx 1.878$, which again is related to the fact that no large "flare" occurs during our timeseries. The distribution for total energy content is flatter than the one for peak activity. We will come back to this point in the final Section.

The average input power (not shown) presents variations of the same order of magnitude occurring on the same timescale as the dissipated power and the total energy content. This feature is due to the fact that the forcing term for the magnetic energy presents a coupling between $F_m(x, y, t)$, which is a smooth function both of time and space, and the magnetic field $B_z$ which on the contrary is highly structured. This coupling is the reason why, no matter what the forcing term in the Faraday equation, the input power is a strongly fluctuating function of both space and time.

The overall dynamics is dominated by an inverse cascade of the vector potential so that, although the forcing contains three to four randomly oriented eddies, the magnetic structure aligns coherently along any one axis of the numerical domain. The trend of the system is therefore to organize itself in well-defined magnetic loops (in the two-dimensional cross-section the loops appear as islands) separated by narrow localized current sheets where intense dissipation episodes occur. The typical lifetime of these current sheets is of the order of the dynamical time scale ($\sim 0.1\tau$).

In Fig. 3 we present snapshots of the local structure of magnetic field, current density, velocity field and vorticity. A current sheet appears clearly in the center of the frame with a maximum value in system units of 82.98, almost one order of magnitude higher than the spatial noise threshold defined below. The maximum value attained by the magnetic field in the box is 14.33 $B_0$, which is much smaller than the large-scale magnetic field $B_0$ we assume to exist in the $z$-direction. From an inspection of the velocity and vorticity structure it appears that reconnection is taking place within the current sheet and that reconnection is the mechanism responsible for the dissipation of magnetic energy.
for its disruption. In fact we observe jets outcoming along the sheet with a typical quadrupole structure of the vorticity. The maximum value of the velocity in poloidal Alfvén velocity units is 2.41 inside the jet, whereas the average velocity throughout the grid is 0.95. The maximum velocity is therefore higher than average and much higher than the assumed photospheric velocity, that in our units is 0.1. It is therefore smaller by almost one order of magnitude than the maximum poloidal Alfvén velocity in the box but exceeds the local value of the poloidal Alfvén velocity around the current sheet.

A statistical analysis of the spatial configurations may be carried out in a fashion analogous to that of the time-series, by constructing a distribution function of spatial dissipation events, i.e., the distribution function of sites with a given current \( J \) as a function of \( J \). Including the data from 120 configurations of the current density in the high resolution 256 × 256 run, we find that for low values of \( J \) the distribution is well fitted by a Gaussian component of the form

\[
N(x) = C_1 \exp\left[-\frac{(x - C_2)^2}{\sigma^2}\right]
\]

where \( x \equiv J \) and \( C_1, C_2, \sigma \) are the fitting constants. For larger values of \( J (\geq 9) \), the distribution diverges from the white noise (gaussian) component and we therefore use this value to define a threshold for spatial events. After having subtracted the background from the spatial current structures we may again construct the distribution function of their magnitudes. The distribution function of \( J^2 \), with \( J \) denoting the

© European Space Agency • Provided by the NASA Astrophysics Data System
current densities that extend above noise, is well approximated by two power laws with different scaling indices, namely $d_1 \approx -2.07 \pm 0.05$ for small values of $J^2$ and $d_2 \approx -2.61 \pm 0.21$ in the high values range. The power law in the low-energy range of the distribution is flatter than in the high-energy range. The existence of such a component could be due to the poor spatial resolution and therefore might not be particularly meaningful.

Having determined the noise threshold for spatial currents, it is of interest to provide an estimate of the surface filling factor $\alpha$, defined as the ratio between the area occupied by the currents above threshold and the total area of the integration box. The quantity $\alpha$ can be directly compared with the analogous quantity $\alpha$, obtained from observations, defined as the fraction of the observed volume that radiates strongly in a particular waveband because of either a different density or a different temperature from its surroundings. Its value ranges roughly between $\sim 0.1$ and $\sim 0.3$ and seems to be fairly well correlated to the average dissipated power. When the dissipated power peaks, the filling factor seems to saturate at a certain level until the dissipated power falls again. This implies that a sharp increase of activity is due to a local increase of the current rather than a broadening or an increase of the number of current sheets.

So far we have investigated the average dissipation properties and the spatial distribution of currents over limited intervals of time. A time-series which combines the two kinds of information is that for the maximum current (which would correspond to the maximum photon count in an individual pixel over the whole field of view as a function of time). Since $J_{\text{max}}$ corresponds to the maximum current density within the spatial configuration at a given instant, it does not correspond to the temporal profile of a single current sheet, but rather may appear at current sheets in different locations at different instants. We have used the same method, as in the case of the average dissipation power, to extract the noise threshold. The noise component may be again well represented by a $\chi^2$-probability function of the form of Equation (1), where we now consider $x \equiv \eta J_{\text{max}}^2$. The best fit occurred for $n = 5$. The noise threshold in this case is roughly estimated to be $E_n \approx 320$.

Subtraction of noise in the time-series originates a total number of 1950 discrete events (that is, a mean number of events approximately equal to 17.1 hr$^{-1}$). The significant difference between this number and that of the average-dissipation events, is due to the fact that the peaks in this last time-series correspond to dissipation in single current sheets, where we have not applied an averaging process.

The distribution functions of events built on the instantaneous maximum current dissipation are given in Fig 4a, for the total energy content, in Fig 4b for peak activity and Fig 4c for event duration. Again one notices that noise reduction has lead to the emergence of power laws, although their extent is slightly reduced with respect to those determined from the average dissipation (Fig. 2). On the average, maximum dissipation peaks are also barely above noise, i.e., $(S/N)_{\text{max}} \approx 1.134$, although in some cases we obtain dissipative events well above the threshold $(S/N)_{\text{max}} \approx 47.96$. The value of the total energy scaling index is quite similar to the one obtained from the average dissipation time-series (within the error bars), while the scaling index of peak activity is rather lower (in absolute values) than the corresponding index from the average dissipation time-series. The distribution function of events' duration on the other hand, is significantly steeper compared to the first case. Comparing Figs 2c and 4c, we notice that the maximum dissipation events are significantly less extended in time (ranging from $\sim 10$ s (0.1 $\tau$), up to $\sim 300$ s ($3\tau$)), indicating that the behaviour of maximum dissipation is in fact much more intermittent compared to that of the average dissipation. This difference should be expected, since we obtain a much bigger number of maximum dissipation events, compared to the case of the average dissipation.

4. CONCLUSIONS

We have performed a statistical analysis of data resulting from an MHD simulation which models the energy release process in a two-dimensional cross-section of a solar coronal loop. By concentrating on the overall long-time behaviour of the system we have shown that magnetically forced MHD turbulence results in an energy release with properties analogous to those observed in the solar corona, namely the
presence of distinct bursts which follow well-defined power laws in terms of total energy content, duration and peak luminosity. We have also shown that such bursts correspond to forced reconnection in localized current sheets, and that such bursts are associated with bipolar jets in which the plasma is accelerated to the (poloidal) Alfvén speed (see Innes et al. 1997 for observational evidence of such jets seen with SOHO), even though the average kinetic energy remains negligible with respect to the magnetic energy.

The interest of the present results lies in the demonstration of the existence of power laws in the coronal response to the photospheric driver, even though the values of the scaling indices will most probably vary significantly with increasing resolution. Another limitation of our results, as mentioned in the Introduction, arises from the two-dimensional nature of our simulations. Within this model it is impossible to store significant amounts of energy in the system, as the turbulent cascade dissipates any input power in an efficient way. In a truly 3D system magnetic helicity is also injected into the system; global conservation properties associated with minimum energy states at given helicity offer much greater possibilities for the dynamics, which are probably necessary to describe the large-scale energy release manifestations on the sun. One should therefore be cautious when comparing the scaling indices resulting from our simulations with those derived from observational data which refer to events involving topologically complex magnetic structures.

With the caveats given above, it is interesting to remark on the scaling index of the spatial-dissipation power law ($\delta \approx -2.61$) obtained in §3. It results from the analysis of the spatial structure of more than one hundred configurations obtained with the highest resolution used and therefore reflects the details of a large number of highly localized (in space and time) energy release episodes. We expect the absolute value of this index to become larger with increasing resolution because of the enhanced capability to describe smaller scales. This value is about twice the value of the indices arising from the temporal evolution of the spatially averaged dissipation and of the maximum dissipation, and this experimental fact may give some insight into the coronal heating via “nanoflare” question. If we interpret the average dissipation in our simulation in terms of events in the same way observers do, the extrapolation below a given energy may lose meaning, since the spatially averaged output over a given region may never fall below a given value, so that a higher spatial resolution is necessary to define and pick up low energy events. On the other hand, our spatial statistics shows precisely that the number of events at small energies does increase considerably with decreasing energy, providing indirect evidence in favour of the nanoflare scenario.

ACKNOWLEDGMENTS

We would like to thank A. Pouquet and C. Chiuderi for many useful discussions. Numerical simulations have been performed on the Cray facilities of ENEL (Pisa, Italy). This work has received partial support by the Greek State Foundation for Scholarships (IKY) and the EEC program ERASMUS.

REFERENCES

Strauss, H. R., 1976, Phys. Fluids, 19, 134