Magnetopause from pressure balance

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Abstract. The shape of the magnetopause and the field due to magnetopause currents are calculated from the requirement that the pressure in the magnetosheath be balanced by magnetic pressure inside the magnetosphere. The field due to magnetopause currents is calculated to be consistent with the iteratively adjusted magnetopause shape. The field due to current systems inside the magnetosphere is taken from the T96 model [Tsiganenko, 1996], which carries information from ~47,000 magnetic field observations. Many different magnetospheric configurations were found for a variety of conditions. Changes in the shape of the magnetopause with varying dipole tilt angle stood out. The magnetotail and the nose (the point closest to the Sun) were found to shift vertically, in opposite directions, for nonzero dipole tilt. The vertical offset of the nose from the Earth–Sun line varied linearly with dipole tilt angle, reaching ~3 \( R_E \) for maximal tilt and having a weak dependence on solar wind pressure. The formation of a secondary stagnation point just above the sunward cusp was indicated for absolute dipole tilts in excess of 13°. The magnitude of the field strength at its local maximum just behind the cusp was determined as a function of dipole tilt angle and the subsolar field strength. Calculated magnetopause shapes and observed magnetopause crossings were found to be consistent when the tilt angle was taken into account. Variations in the latitude of the magnetic cusps with dynamic pressure, interplanetary magnetic field \( D_p \) and dipole tilt were reasonably consistent with observed variations in the latitude of the particle cusp.

1. Introduction

Magnetopause shapes and the corresponding model magnetic fields are adjusted in order to satisfy pressure balance across the magnetopause. This yields information on the magnetopause size and shape under varying conditions as well as improved knowledge of the magnetic field in the vicinity of the magnetopause. A previous study [Sotireshis, 1996] (hereinafter referred to as paper 1) performed a similar analysis but restricted itself to axisymmetric magnetopause shapes and used a less sophisticated magnetic field model [Tk7We [Pereida et al., 1993]]. In this study the magnetopause is permitted to assume a more general shape described by a \( 61 \times 19 \) (or \( 61 \times 37 \)) grid of points. Also, a more sophisticated magnetic field model based on the T96 model [Tsiganenko, 1996] is used. Rather than repeat the extensive introductory material from paper 1 regarding the general method, we will endeavor here to be brief (see paper 1 for additional details).

The method applied here and in paper 1 is conceptually equivalent to that of Mead and Board [1964]. The magnetopause shape and the field due to magnetopause currents are iteratively adjusted until the magnetic pressure just inside the magnetopause is in balance with the pressure in the magnetosheath as given by the Newtonian approximation. Mead and Board [1964] include only the dipole field and the field due to magnetopause currents, so their magnetopause shape depends only on solar wind dynamic pressure. Here we include the effects of the cross-tail, ring, and Birkeland currents, as they depend on solar wind conditions through the T96 model. The resulting configurations are qualitatively different from those of a confined dipole, being vertically smaller on the dayside and having much larger magnetotails. We show the dependence of this shape on solar wind variables and dipole tilt angle and compare it with observed magnetopause crossings.

Other studies have made use of pressure balance across the magnetopause in order to study magnetospheric configurations allowing for the cross-tail current. Uniti and Atkinson [1968] and Atkinson and Uniti [1969] used complex analysis to find two-dimensional magnetotail solutions. Petrinec and Russell [1993, 1996] used observations of the loxoh field strength and the Newtonian approximation to remotely sense the flaring angle of the magnetopause. The magnetotail radius as a function of downstream distance was then deduced using the flaring angle and a fit to dayside magnetopause crossings.

2. Method

The magnetopause shape is described by a grid of points (Figure 1). This shape is initialized as an axisymmetric ellipsoid from Sibeck et al. [1991]. First the field model is modified to fit the surface described by this grid. Then the magnetic pressure is compared with the shear pressure, and the locations of the points on the grid are adjusted toward pressure balance. This procedure is repeated iteratively, alternating the field modification with the grid adjustment approximately 10 to 20 times. The resulting configurations generally satisfy pressure balance to better than 1% for 70% of the points on the boundary and to better than 2% for 80% of the boundary points (the region where the plasma sheet meets the boundary is excluded from this ratio).
Figure 1. The $61 \times 19$ grid used to represent the magnetopause for zero dipole tilt. GSM coordinates are used with distances given in Earth radii ($R_E$).

Since the field models that we consider are all dawn–dusk symmetric, we only have to solve the problem over half of the magnetopause. We choose the dusk side. For zero dipole tilt, north–south symmetry holds, and we need consider only one quarter of the magnetopause surface. In this case we solve in the north–dusk quadrant. The grid points are spaced every 5° of rotation about the $x$ axis. In the orthogonal direction, they are spaced every 4.3° on the dayside from the $x$ axis to the terminator and on the nightside from 1 $R_E$ in $x$. The result is a $61 \times 19$ grid for zero dipole tilt and a $61 \times 37$ grid otherwise.

The T96 model field is modified to fit an arbitrary magnetopause shape by adjusting it to satisfy an appropriate boundary condition at every point of the grid. The condition of pressure balance is applied only for every other point in $x$. The intervening points are determined by interpolation. This is a smoothness constraint that was found to improve the solutions on the dayside for nonzero dipole tilt where steep gradients in the T96 model's shielding field can introduce small-scale irregularities.

For comparison we calculate configurations with just a dipole field. In this case we truncate the magnetopause grid at $x = 20 R_E$, since tailward of this point the magnetosheath pressure cannot be balanced by the very weak magnetic pressure. Mead and Beard [1964] did not encounter this problem because they used a less realistic form of the Newtonian approximation, in which the magnetosheath pressure goes to zero in the distant tail. (We use 0.03 nPa for the distant sheath pressure, which corresponds to a field of 9 nT.)

The calculational scheme in which magnetic pressure inside the magnetosphere is balanced against pressure in the sheath as given by the Newtonian approximation cannot deal with two situations. First, in the region where the plasma sheet meets the magnetopause, the pressure on the magnetospheric side of the magnetopause can no longer be approximated by just the magnetic pressure. Since we do not have a model for the plasma pressure, we simply interpolate a smooth surface over this region from above and below. Second, in the vicinity of the cusp, the magnetopause collapses inward to form a funnel-like region of magnetosheath-like plasma that impinges directly on the ionosphere. Our method has no hope of duplicating this finely detailed structure. It does find an indentation in the magnetopause boundary corresponding to the high-altitude extension of this structure. The reader should keep in mind that such an indentation corresponds to the expected location of such a funnel.

The Newtonian approximation (described in the next section) provides an analytic form for the sheath pressure as a function of the position of a grid point, but the magnetic pressure is available only from the previous iteration. For this reason we assume a power law dependence for the magnetic pressure as the grid point deviates from its previous position. Different powers were used in different regions in an attempt to balance the speed with which equilibrium is approached against the tendency to oscillate around that equilibrium position once in its vicinity. Additionally, damping was applied by setting the new point to a weighted average of the newly calculated position and the previous position. The slowest approach and most damping were needed near the subsolar point, while the distant tail required the least. Smoothness constraints were also applied to aid in convergence. Proper optimization of these many technical details was important in achieving pressure balance in a reasonable number of iterations. Poorly optimized schemes could cause the solution to either diverge or approach equilibrium too slowly. Once convergence is achieved, there is very little difference among the solutions that result from different schemes.

2.1. Magnetsheath Pressure

To leading order, the flow of the solar wind around the magnetosphere can be characterized hydrodynamically, as a supersonic flow impinging on a blunt obstacle, which is the magnetosphere [Spreiter et al., 1968]. As the flow decelerates upstream of the magnetopause the energy associated with that flow is converted into thermal energy. Near the subsolar magnetopause a stagnation point forms, where the pressure is of the same order as the upstream dynamic pressure

$$P_{\text{stag}} = P_{\text{dyn}} = \rho_u v_u^2$$

where $\rho_u$ and $v_u$ are the upstream density and velocity, respectively. Along the magnetopause, flow speeds increase with distance from the stagnation point, gradually approaching their former value $v_u$ in the distant magnetosheath. Similarly, the pressure in the magnetosheath decreases, gradually approaching $P_u$ the pressure in the upstream solar wind, which is usually 1 or 2 orders of magnitude less than the dynamic pressure.

(The hydrodynamic flow described above is merely a tool used to approximate the total pressure at the magnetopause. The thermal pressure in this scenario is a surrogate for the actual total pressure (the sum of the thermal and magnetic pressures). The total pressure increases as the flow decelerates, first with a large jump at the bow shock and then more gradually as the stagnation point is approached. If a plasma depletion layer forms as magnetic flux piles up near the stagnation point, the actual thermal pressure may fall considerably, but the total pressure will not.)

In hydrodynamic flows as described above the pressure in the magnetosheath at the obstacle surface can be approximated by an empirical relation called the Newtonian approximation:

$$P = K P_{\text{dyn}} \cos^2 \theta + P_u$$  \hspace{1cm} (1)

where $\theta$ is the angle between the upstream solar wind velocity and the surface normal. The Newtonian approximation is normalized by choosing $K$ to agree with the exact hydrodynamic result for pressure at the stagnation point. The approximation in applying this result to the magnetosphere is in the extent to which the hydrodynamic result for pressure at the stagnation point approximates the actual value and in the interpolation be-
tween this value $P_{\text{magn}} = K (P_{\text{sw}} + P_w)$ and the pressure in the distant magnetosheath $P_w$. Under most conditions this approximation does quite well, although for low Mach numbers it is less accurate.

An encouraging result from paper 1 is that the final configuration is insensitive to inaccuracies in the Newtonian approximation's interpolation between $P_{\text{magn}}$ and $P_w$. Modest changes in the $\cos^2 \theta$ dependence of the sheath pressure made little difference in the final solution, both because those changes are small compared with the wide range of variation between $P_{\text{magn}}$ and $P_w$ and because the magnetic pressure is a strong function of the magnetopause shape that we are varying $P_M \sim \theta^2 \sim r^{-d}$ (lobe) or $r^{-5}$ (dasyide). A 40% change in the sheath pressure where the Newtonian approximation seems least accurate resulted only in a 0.2 $R_E$ difference in magnetopause position at that location.

2.2. Magnetic Field Model

The magnetic field model used to obtain the magnetic pressure,

$$P_M = B^2 / 2\mu_0$$

is a modified version of the T96 model [Tryggestad, 1996]. We superpose an additional field on this model such that its magnetopause conforms to that specified by our grid. In this way, when the grid is moved inward, the field (confined now to a smaller region) increases, and the converse occurs when the grid is moved outward. First we describe the T96 model, and then we describe the method by which we modify it to fit an arbitrary magnetopause.

The T96 model is used by our procedure as input. It incorporates explicit forms for the cross-tail, ring, and Birkeland region 1 and 2 currents. In addition, it has a confinement field to represent the effect of magnetopause currents and an interconnection field permitting a small but nonzero field normal to the magnetopause. Amplitudes of the various current systems are controlled by the solar wind dynamic pressure $P_{\text{sw}} = \rho_{\text{sw}} v_{\text{sw}}^2$, the interplanetary magnetic field (IMF) $B_y$ and $B_z$ components, and $D\text{st}$. IMF dependence is through the term

$$\gamma = \frac{n^{1/2}}{\nu_{\text{sw}} (B_y^2 + B_z^2)^{1/2}} \left| \sin(\theta/2) \right|$$

(2)

where $\theta = \tan^{-1}(B_y/B_z)$. The degree to which the magnitude of each current system depends on each of these parameters is determined by coefficients that are obtained from the fit to the data.

The T96 model field is confined to a prescribed magnetopause shape that is based on an ellipsoidal fit to observed crossings [Sibeck et al., 1991] but is continued by a cylinder tailward from the point of its maximum girth. This confining shape scales as a function only of the solar wind dynamic pressure. It is this shape that we hope to improve upon by superposing an additional confinement field.

In this study, the T96 model field is made to conform to an arbitrarily shaped magnetopause grid by the superposition of an additional scalar potential field. This is equivalent to altering the magnetopause current implied by the model field [Sotirelis et al., 1994]. The scalar potential field that we superpose is expanded in parabolic harmonics (used by Stern [1985] for a parabolic magnetopause):

$$\Phi = \sum_{m=0}^{M} \sum_{n=-1}^{N} \left( A_{nm} \cos m\phi + B_{nm} \sin m\phi \right)$$

where $A_{nm}$ and $B_{nm}$ are the coefficients determined by the fit.

**Figure 2.** Field line configurations for northward interplanetary magnetic field (IMF) ($B_y = 5$ nT, $B_z = 0$, $P_{\text{sw}} = 2$ nPa, and $D\text{st} = -10$ nT): (a) the unmodified T96 field, (b) the adjusted configuration that maintains the normal component of $B$, and (c) the adjusted configuration that sets the normal component of $B$ to zero.
The coefficients $A_{nm}$ and $B_{nm}$ are found by least squares minimizing the boundary condition on the magnetopause grid. In paper 1 the boundary condition that was used was to set the normal component of $B$ to zero. The T96 model has an open magnetopause, however. While this small amount of open flux should not make much difference, since the magnetopause is still almost closed, we prefer to change the T96 model as little as possible. The case of southward IMF is relatively straightforward. We simply maintain the normal component of $B$ unchanged.

For northward IMF the situation is less clear. In this case, the flux crossing the T96 model's magnetopause does not connect to the Earth, but takes the form of lobe field lines that connect directly to the magnetopause across the high-latitude magnetopause (Figure 2a). Such field lines may or may not occur, but their existence in this model is probably spurious. For

![Figure 3](image_url)

**Figure 3.** Contour plots of the magnetic field magnitude (in nanoteslas) in the noon–midnight meridian for southward IMF (IMF $B_z = -5$ nT, $B_y = 0$, $P_{dyn} = 2$ nPa, and $Dst = -10$ nT): (a) the total field, and (b) just the superposed adjustment field.

The parabolic coordinates $(\lambda, \mu, \phi)$ are given by

$$
\lambda = \sqrt{r^2 + (x - x_{rm})^2} \\
\mu = \sqrt{r^2 - (x - x_{rm})^2} \\
\phi = \tan^{-1}(y/l),
$$

$(x, y, z)$ are GSM coordinates, and $r$ is the distance from the focus ($x_{rm}, 0, 0$).

Each mode is permitted to have its own wave number $k_{nm}$ and to have a separate location for the focus ($x_{rm}, 0, 0$). The wave numbers $k_{nm}$ are then selected so that there are nodes at the subsolar point and at the distant edge of the magnetopause grid and $n-1$ nodes in between.

It is not always necessary to make use of all this generality, but confining the T96 can be a bit problematic. The T96 model's shielding field is a superposition of components whose magnitudes increase exponentially with $x$. The resulting high-frequency behavior can cause problems in the vicinity of the subsolar magnetopause, where sharp gradients reduce the quality of our confinement. To minimize this effect, $x_{rm}$ is selected to reduce the amplitude of the higher-frequency modes in this region (higher frequency modes have larger $x_{rm}$, typically $x_{rm} \sim 10 R_E$). We have high confidence in the large-scale features of our solutions, but unphysical small-scale features are sometimes present.

![Figure 4](image_url)

**Figure 4.** Field line configurations for southward IMF (IMF $B_z = -5$ nT, $B_y = 0$, $P_{dyn} = 2$ nPa, and $Dst = -10$ nT): (a) the unmodified T96 field, and (b) the adjusted configuration.
this reason we require the magnetopause to be closed for northward IMF (Figure 2c). Regardless, the difference between the two cases is small (compare Figures 2b and 2c). It is not very likely that the actual magnetopause is completely closed, but as long as the flux through it is less than the flux crossing the T96 model's magnetopause, our results should not be too far off. If, however, the magnetopause is very open in some region (e.g., behind one of the cusps), then our solution would represent an inner bound in that region.

The T96 model has a number of free parameters that are determined by a fit to ~47,000 magnetic field observations. A legitimate concern is that by modifying the model we might ruin its correspondence with these observations. We confirm that this is not the case by verifying that the field we superpose (Figure 3b) is small compared with the model field (Figure 3a) except near the magnetopause. This is acceptable since the magnetopause is where we are trying to improve the model and where the observational coverage is thin. In this way we have effectively obtained the field due to currents inside the magnetosphere from the observations to which the T96 model was fit while allowing the field due to magnetopause currents to vary with the magnetopause shape.

3. Results

3.1. The Magnetopause for Zero Dipole Tilt Angle

In Figure 4 we demonstrate our results with field line plots for the T96 model (for \( P_{\infty} = 2 \) nPa, \( B_z = -5 \) nT, \( B_y = 0 \), and \( Dst = -10 \) nT) both before and after being adjusted into pressure balance (for \( B_z = 5 \) nT, refer to Figure 2). GSM coordinates with distances given in Earth radii \( (R_E) \) are used throughout. Comparison with the dipole result depicted in Figure 5 shows a distinct difference between the two. The addition of the T96 currents to the dipole field inflates the magnetotail significantly and brings the cusps farther equatorward, resulting in a vertically smaller dayside magnetosphere.

Figure 6. Magnetopause outlines for dynamic pressures of 1, 2, 4, and 8 nPa from outside to inside: (a) in the noon-midnight meridian and (b) 30° above the equatorial plane (i.e., 30° of rotation about the x-axis). In the top half of each plot, IMF \( B_z = +5 \) nT, and in the bottom half IMF \( B_z = -5 \) nT (for both halves, IMF \( B_y = 0 \) and \( Dst = -10 \) nT).

In Figure 6 we provide families of magnetopause shapes for varying solar wind dynamic pressures in the noon-midnight meridian (Figure 6a) and rotated 30° above the equatorial plane about the x-axis (Figure 6b). We do not report the shape in the equatorial plane itself, since in the tail it is the result of an extrapolation from higher latitudes. In Figure 7 we provide a similar family of plots but for varying IMF \( B_z \). We present only one curve for purely northward IMF because the IMF dependence of the T96 current systems is through (2), which is zero, so there is little difference between them. The only T96 model variation with the magnitude of a purely northward IMF is due to the interconnection field and is probably not very meaningful.

3.2. The Magnetopause for the Case of a Tilted Dipole

In Figure 8, field line plots are shown at maximal negative tilts (~33°) for a dipole field confined by a 2-nPa solar wind

Figure 5. The field of a confined dipole for \( P_{\infty} = 2 \) nPa. This is analogous to the work of Mead and Beard [1964].
Figure 7. Magnetopause outlines for varying IMF $B_z$, in the noon–midnight meridian are shown in the top half of the plot, and those for 30° above the equatorial plane are shown in the bottom half. Other conditions were IMF $B_x = 0$, $P_{sw} = 2$ nPa, and $Dst = -10$ nT.

(Figure 8a), for the T96 field (Figure 8b), and for our result (Figure 8c). Our result permits an asymmetric, pressure-balanced magnetopause together with a stretched and inflated tail. In Figure 9, noon–midnight meridian sections of the magnetopause are shown for different dipole tilt angles.

For varying dipole tilt, two effects are readily apparent. The entire magnetotail shifts vertically, and the nose (the sunwardmost point) shifts vertically in the opposite direction. These shifts are understandable since the near-Earth magnetic field configuration turns with the dipole tilt. When the field shifts, the location of the equilibrium shifts, causing the nose to shift one way and the tail to shift the other.

The amount by which the nose shifts depends on the dynamic pressure and appears insensitive to IMF $B_z$, within the accuracy to which we are able to determine it. The uncertainty in our determination of the shift for the T96 model is approximately 0.2–0.4 RE, which is greater than for the confined dipole (0.1–0.2 RE) due to the small-scale irregularities alluded to previously. For the case of the confined dipole the shift is roughly half that seen with the T96 model. Presumably, this is because of the presence of cross-tail and Birkeland currents, which move the cusps farther equatorward, causing the dayside magnetopause to be more sensitive to dipole tilt effects.

The displacement of the nose from the Earth–Sun line as a function of dipole tilt angle is shown in Figure 10a. The behavior is linear, with a slope of $-0.09$ RE per degree of tilt. We show only negative tilts but have verified that this linear behavior holds for positive tilts. In Figure 10b the displacement of the nose from the Earth–Sun line at maximal negative dipole tilt (−35°) for different solar wind dynamic pressures is shown, for both the T96 model (plus signs) and for a confined field dipole (crosses). The tilt angle dependence at each pressure can be taken as linear.

For absolute dipole tilts in excess of −15° the postcusp magnetopause in the hemisphere tilted toward the Sun stands

Figure 8. Field line configurations for dipole tilt $\Psi = 35^\circ$ and $P_{sw} = 2$ nPa: (a) the field of a confined dipole, (b) the unmodified T96 field, and (c) the adjusted configuration. For Figures 8b and 8c, IMF $B_z = -5$ nT, $B_y = 0$, and $Dst = -10$ nT.
flow slows significantly, whenever the absolute tilt exceeds ~15°. This prediction is made within the framework of the Newtonian approximation, wherein the maximum pressure exerted on an obstacle is achieved through the nearly complete deceleration of a gasdynamic flow, converting bulk flow energy into thermal energy.

The magnitude of the local magnetospheric field maximum just behind the cusp is a function of the dipole tilt angle (Figure 11). One might expect the merging rate for northward IMF (when merging is expected behind the cusp) to depend on the dipole tilt angle since the magnitude of the magnetopause current is proportional to the magnetospheric field. An additional degree of correlation might be expected since the degree to which the postcusp region stands out into the sheath flow and forms a secondary stagnation region is measured by the pressure, which is proportional to the square of the magnetospheric field.

3.3. Comparison with Observed Magnetopause Crossings

In order to verify these tilt angle effects we compare with observed magnetopause crossings. Two sets of magnetopause crossings are used: a collection from several sources assembled by Sibeck et al. [1991] populating mostly lower latitudes and a set of high-latitude crossings by the HAWkeye spacecraft identified by S. Boardman et al. (An empirical model of the high latitude magnetopause, submitted to Journal of Geophysical Research, 1998; hereinafter referred to as submitted manuscript). In Figure 12, field line plots for $P_{dynam} = 2 \text{nPa}$, IMF $B_z = -5 \text{nT}$, and $Dst = -10 \text{nT}$ are overlaid with the crossings from Sibeck et al. [1991] (crosses) and from S. Boardman et al. (submitted manuscript, 1998; plus signs). The crossings from Sibeck et al. [1991] were tagged with hourly averages of IMF $B_z$ and dynamic pressure. The crossings from S. Boardman et al. (submitted manuscript, 1998) were tagged with hourly averages of IMF $B_z$ and 5-min averages of solar wind pressure. Crossings with dynamic pressures between 1 and 4 nPa were transformed to 2 nPa assuming a 1/6 power scaling with pressure. The crossings were selected for dipole tilt between $-10^\circ$ and $10^\circ$ for comparison with the zero tilt case (Figure 12a) and between $\pm 20^\circ$ and $\pm 30^\circ$ for comparison with the $-25^\circ$ tilt case (Figure 12b). Crossings for positive tilt were reflected into the opposite hemisphere. The comparison is quite favorable. The
3.4. The Cusp

A key feature of these results is the indentation in the vicinity of the high-altitude cusp and its dependence on dipole tilt (Figure 13). This indentation is not present in the T96 model, and for the confined dipole it is at higher latitudes and less pronounced. Observations made in this vicinity should be interpreted accordingly.

The low altitude location of the field line that threads the magnetic cusp is provided for three different values of IMF $B_z$ in Figure 14. The latitude is given as a function of solar wind dynamic pressure in Figure 14a and as a function of dipole tilt angle in Figure 14b. In Figure 14b the latitude for the cusp that is tilted toward the Sun is given by the branch labeled “toward,” and similarly, the branch for the cusp tilted away is labeled “away.” The differences between these results and the unmodified T96 model are small, varying from a few tenths of a degree for low values of dynamic pressure and dipole tilt up to $-1^\circ$ for large values of dynamic pressure and dipole tilt (not shown).

Comparisons with the statistically determined latitude of the particle cusp may be offset from our results by up to $-2^\circ$ depending on the nature of the comparison. Since comparison is being made between the magnetic cusp and various measures of the location of the particle cusp, such offsets are not surprising. Changes in cusp latitude with variations in dipole tilt angle, dynamic pressure, and IMF $B_z$ are, however, in reasonable agreement. Newell and Meng [1989] find that the low-latitude edge of the particle cusp moves $4^\circ$ when dipole tilt changes from $-30^\circ$ to $30^\circ$ (see the dashed line in Figure 14b), which is not too far from the $3^\circ$ variation that we find. Newell and Meng [1994] find the low-latitude edge of the statistical cusp to be $77.5^\circ$ for $<P_{dyn}> = 1.5$ and $76.2^\circ$ for $<P_{dyn}> = 6$ (taken from their Figure 1 and shown with crosses connected with a dashed line in our Figure 14a). They see a change of $1.3^\circ$ between $<P_{dyn}> = 1.5$ nPa and $<P_{dyn}> = 6$ nPa, while we get a $\sim 1^\circ$ change (depending on IMF). The absolute comparison between these results is not clear since we have not performed a properly weighted average over all IMF conditions in our result. One would expect the result of such an average to fall between our IMF $B_z > 0$ and IMF $B_z = -10$ nT curves. Since the results with which we compare lie near the center of this range, any disagreement is not likely to be very large.

In order to compare under different IMF conditions it must be noted that IMF $B_z$ was set to zero in our calculation so that we could take advantage of dawn-dusk symmetry. Since the magnitudes of most of the current systems in the T96 model are at least partly controlled by (2) and $<\text{IMF} B_z>$ is not zero, we compare our IMF $B_z = -5$ nT result with the statistical result for IMF $B_z = -3.8$ nT (this assumes $<\text{IMF} B_z> \sim \text{IMF} B_z$). By comparing with the low-latitude edge of the particle cusp from Newell et al. [1989], we find that we agree with the cusp varying little for northward IMF $B_z$, and moving $-3^\circ$ equatorward for $<\text{IMF} B_z> = -3.8$ nT. Their results are shown in Figure 14a by the solid circle for small positive IMF $B_z$ and the open circle for $<\text{IMF} B_z> = -3.8$ nT, which should correspond to our IMF $B_z > 0$ and IMF $B_z = -5$ nT results, respectively. Both of these results are offset from ours by almost $2^\circ$.

4. Summary

Magnetospheric configurations that respect pressure balance across the magnetopause were found for a variety of conditions by modifying the T96 model. The magnetosphere was found to
shape used in the T96 model. Many such configurations were presented for general reference.

Several tilt angle effects were noted. The sunwardmost point of the magnetosphere (the nose) and the magnetotail shift vertically, in opposite directions, by up to 3 to 4 $R_E$ for nonzero dipole tilt. The vertical offset of the nose was found to vary linearly with dipole tilt angle, and the slope of this variation was found to depend weakly on solar wind dynamic pressure. The magnitude of the local magnetospheric field maximum just behind the cusp was determined as a function of dipole tilt angle and the subsolar field strength. Since the pressure behind the cusp that is tilted toward the Sun is close to subsolar values when absolute dipole tilt angle exceeds 15°, the formation of a secondary (pseudo)stagnation point is implied (within the framework of the Newtonian approximation and gasdynamic flow, most of the energy from the bulk motion is converted into pressure, but the flow probably does not truly stagnate).

Calculated magnetopause shapes for different dipole tilt angles were compared with observed magnetopause crossings. Both the calculated shapes and the crossings showed the tail and the dayside magnetosphere moving vertically, in opposite directions, for nonzero dipole tilt. The latitudes of the magnetic cusps within the calculated field models were found to vary in a manner consistent with observed variations in the location of the particle cusp.

The results presented here are the result of balancing the sum of the thermal and magnetic pressures in the

**Figure 13.** A close-up of the magnetic cusp for different dipole tilt angles: (a) $\Psi = 25^\circ$, (b) $\Psi = 0^\circ$, and (c) $\Psi = -25^\circ$. Other conditions were IMF $B_z = 5$ nT, $B_y = 0$, $P_{\text{dyn}} = 2$ nPa, and $DST = -10$ nT.

**Figure 14.** Absolute magnetic latitude of the field line that threads the magnetic cusp as a function of (a) dynamic pressure and (b) dipole tilt angle.
magnetosheath, estimated by the Newtonian approximation to gasdynamic flow, against the magnetic pressure inside the magnetosphere. The magnetospheric magnetic pressure is calculated by using the current systems of the T96 model together with self-consistently calculated magnetopause shapes and currents. Any error in estimating the sheath pressure could be significantly reduced by using an MHD estimate to the total pressure at the stagnation point, as our technique does not appear to be sensitive to inaccuracies in the $\theta$ dependence of the Newtonian approximation. The results presented here inherit the parameterization of the T96 model and would be improved by any advances in the parameterization of future models. High-frequency components in the T96 model field caused some small-scale artifacts in the results presented here, although the large-scale features are robust. If future empirical models have less high-frequency content, then the small scale features that result from our procedure will be more accurate. We prescribed a closed magnetopause for northward IMF. With better guidance as to the actual normal component our results would be more accurate, although probably not very different.

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